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Geometrical Properties of 3-D Images and Its Uses in 3-D Image Processing

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Abstract: This paper discusses geometrical properties of the 3-D images and applications in 3-D image processing. The main focus is given to neighborhood geometric property. This paper also discusses three dimensional grid and its properties. 3-D Mathematical erosion and dilation is taken as an example to explain the use of neighborhood.

Keywords: Three Dimensional Grid, 3-D Images; 3-D Image Processing, 3-D Image Analysis.

I. INTRODUCTION

Three-dimensional (3-D) objects are now very widely used in a numerous daily applications such as CAD/CAM, games, cinema, animation, virtual reality, medical imaging, archaeology, culture, and meteorology etc. 3-D shape analysis or analytics play a vital role in classification, Computation, measurement and retrieval of 3-D images or 3-D models. As a result, many researchers are working on shape analysis and classification of 3-D images with a goal to analyze the 3-D objects.

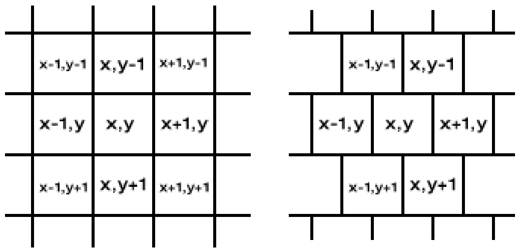
To process and Investigate 3-D shapes there is a need to understand the basic geometrical properties of 3-D objects. The primary objective of this paper is to review various geometrical properties of 3-D images and their use in shape analysis of 3-D images.

The Section 2 of this paper is a quick introduction of various three dimensional grids and their notations. The Section 3 examines important geometrical properties of 3-D images and Section 4 discusses the use of few geometric properties in the processing 3-D images.

II. THREE DIMENSIONAL GRID

2-D images can be represented in a rectangular grid or in a hexagonal grid [1][2] [4][6]. Fig 1 represents these grids and correlating pixel positions. The number of neighboring pixels for the basic pixel, in rectangular grid is 8 and for the hexagonal grid is 5. It is possible to have ambiguous diagonal configurations in Hexagonal Grid. It is not easy to see that this structure is based on the affluent possible rotation group on a grid. It is very easy to correlate the associated algorithms also as it is very simple.

In general, the repeated grid is called as tessellation. With respect to N dimensional case when N≥3, tessellation is also known as honeycombing. The basic honeycombing exists in the case of N=3, rectangular or hexagonal.

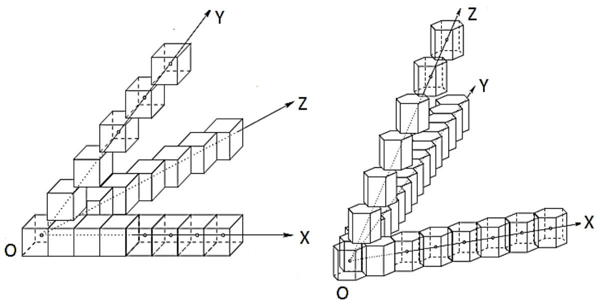


(a) (b)

Fig 1: (a) Rectangular grid (b) Hexagonal grid

Fig 2 (a) represents rectangular 3-D grid and Fig 2 (b) represents hexagonal 3-D grid, In which O is called voxel and X origin, Y, and Z are directions.

Let $\mathbb{Z}$ indicate the set of integers and $\mathbb{R}$ Indicate the set of real numbers. $\mathbb{Z}$ is a proper subset of $\mathbb{R}$. $\mathbb{R}^n$ Indicates n-dimensional Euclidean space and $\mathbb{Z}^n$ Indicates the set of all discrete lattice points embedded in $\mathbb{R}^n$. Any grid point is represented as$(x, y, z) \in \mathbb{Z}^n$. Fig 3 represents orthogonal planes of the Euclidean space $\mathbb{R}^3$.



(a) (b)

Fig 2: 3-D grid (a) Rectangular (b) Hexagonal

The cellular space $\mathbb{C}^n$ is a union of cells, which are either cubes, edges, vertices or faces. The 3-D digital geometry deals with geometrical properties of "3-D digital images", they are sets of points or cubes of the digital

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space $\mathbb{Z}^n$ or $\mathbb{C}^n$ [1]. In generic, computer displays are manufactured as rectangular grids having each cell is an array of square pixels.

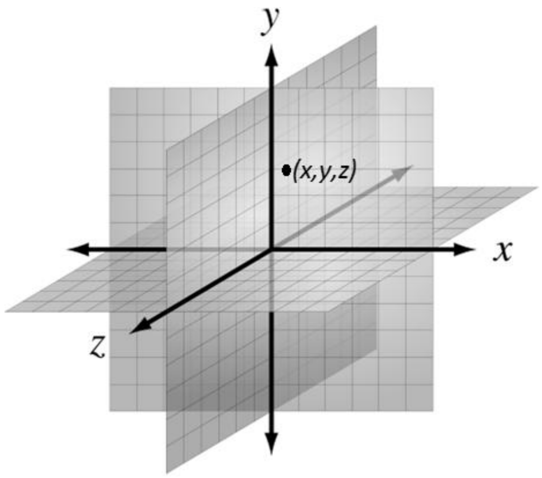
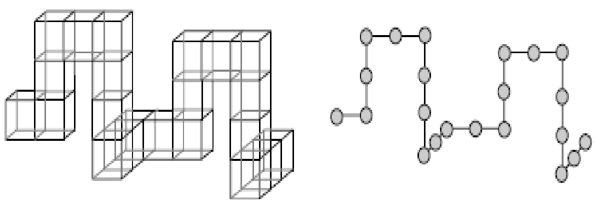


Fig 3: Orthogonal planes of Euclidean space $\mathbb{R}^3$.

In generic way, the rectangular 3-D grid consists of different types, cubic grid, cubic centered grid and face centered cubic grid. The Alternatives to the square/rectangular cubic grids, hexagonal grids are also well-considered for displaying either 2-D or 3-D images, whose cells are regular hexagons. Fig 4 represents a grid cell model and a grid point model [2] [4].



(a) (b)

Fig 4: (a) Grid Cell model (b) Grid Point model

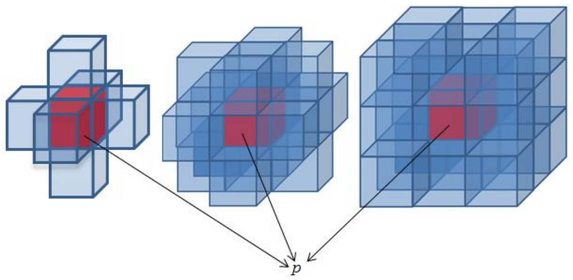
III. GEOMETRICAL PROPERTIES OF 3-D IMAGES

A 3-D digital image consists of a set of grid cells in $\mathbb{Z}^n$ and there prevails proximity relation among them. A digitized 3-D image is a union of number of voxels in a 3-D grid. These voxels may have some intensity/color assigned to them. The number of voxels depend on the size of a 3-D image. For example, a 3-D image of size 256x256x256, the number of voxels is 2563. Some of the geometrical properties of 3-D images are 'Neighborhood', 'Adjacency', 'Path, Connectivity and Connected Component', 'Borders and Interiors', 'Arcs and curves', 'Path length', 'Path-wise distance', 'Intrinsic distance', 'Surface Area, eccentricity, radius and diameter' and 'Convexity and straightness' [3]. In this paper Let us briefly examine few of the geometrical properties such as Neighborhood and Adjacency. We utilize them in the formulation of various conceptual, computational tools and algorithms.

Neighborhood

Let's consider a 3-D rectangular grid of voxels to represent 3-D images. Fig 5 represents 8-neighborhood, 18-neighborhood, and 26-neighborhood in a 3x3x3 rectangular array of voxels.

In effect, most frequently neighborhoods that are considered for processing 3-D digital images are 6-neighborhood, 8-neighborhood, 12-neighborhood, 18-neighborhood and 26-neighborhood [3]. Fig 6 represents a 3X3X3 grid of labeled voxels.



(a) (b) (c)

Fig 5: (a) 6-neighborhood (b) 18-neighborhood (c) 26-neighborhood

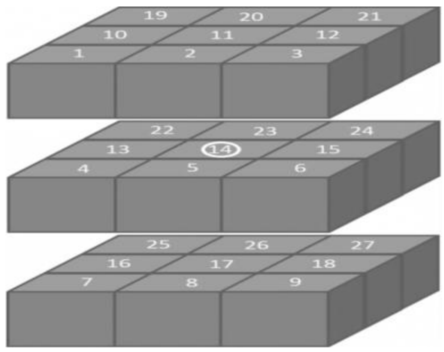


Fig 6: Labeled voxels in the 3X3X3 neighborhood.

Fig 6 represents the equivalent numbering of the voxels in 3x3x3 rectangular grid. Consider the following neighborhoods [3][7].

$$S_1 = \{5, 11, 13, 15, 17, 23\}$$
$$S_2 = \{2,4,6,8,10,12,16,18,20,22,24,26\}$$
$$S_3 = \{1, 3, 7, 9, 19, 21, 25, 27\}$$

S_1 consists of voxels with one unit distance from the central voxel14.

S_2 consists of voxels with $\sqrt{2}$ unit distance from the central voxel 14.

S_3 consists of voxels with $\sqrt{3}$ unit distance from the central voxel 14.

Let us examine a voxel pair (u,v) where $u=(x_i,y_i,z_i)$ $v=(x_j,y_j,z_j)$. Currently, voxel v is said to be n-neighborhood in place if it belongs to any one of the following:

- **6-neighborhood [$N_6(v)$]:** The 6-neighborhood $N_6(v)$ contains of six voxels whose location in the 3-D grid from

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v is by one unit distance. This is clearly demonstrated in Fig 5 (a) by means of a central red color voxel p.Fig 5.(a) represents the 6-neighborhood property of the neighborhood. The neighborhood of all voxels of v fulfills the following condition [3][7]:

$$|x_i - x_j| + |y_i - y_j| + |z_i - z_j| = 1 \quad (1)$$

Now, $N_6(v) = S_1 = \{5, 11, 13, 15, 17, 23\}$

• **12-neighborhood [$N_{12}(v)$]:** The 12-neighborhood $N_{12}(v)$ consists of twelve voxels whose locations in the 3-D grid distant from v by $\sqrt{2}$ distance. This is clearly demonstrated in Fig 8 (d) by means of a central red color voxel p. Fig 8 (d) represents the 12-neighborhood property of the neighborhood. The neighborhood of all voxels of v fulfills the following condition [3][7]:

$$|x_i - x_j| + |y_i - y_j| + |z_i - z_j| = 2 \quad (2)$$

Now,$N_{12}(v) = S_2 = \{2,4,6,8,10,12,16,18,20,22,24,26\}$

• **8-neighborhood [$N_8(v)$]:** The 8-neighborhood $N_8(v)$ consists of eight voxels whose locations in the 3-D grid distant from v by $\sqrt{3}$ distance. This is clearly demonstrated in Fig 8 (e) by means of a central red color voxel p. Fig 8 (e) represents the 8-neighborhood property of the neighborhood. The neighborhood of all voxels of v fulfills the following condition [3][7]:

$$|x_i - x_j| + |y_i - y_j| + |z_i - z_j| = 3 \quad (3)$$

Now, $N_8(v) = S_3 = \{1, 3, 7, 9, 19, 21, 25, 27\}$

• **18-neighborhood [$N_{18}(v)$]:** The 18-neighborhood $N_{18}(v)$ consists of six voxels whose locations in the 3-D grid distant from v in only one unit distance and twelve voxels whose locations in the 3-D grid distant from v by $\sqrt{2}$ distance. This is clearly demonstrated in Fig 5 (b) by means of a central red color voxel p. Fig 5 (b) represents the 18-neighborhood property of the neighborhood. The neighborhood of all voxels of v fulfills the following condition [3][7]:

$$N_{18}(v) = N_6(v) \cup N_{12}(v) \quad (4)$$

$$1 \le |x_i - x_j| + |y_i - y_j| + |z_i - z_j| \le 2 \quad (5)$$

(or)

$$\text{Now, } N_{18}(v) = S_1 \cup S_2 \quad (6)$$

• **26-neighborhood [$N_{26}(v)$]:** The 26-neighborhood $N_{26}(v)$ consists of six voxels whose locations in the 3-D grid distant from v in one unit distance, twelve voxels whose positions in the 3-D grid distant from v by $\sqrt{2}$ distance and eight voxels whose locations in the 3-D grid distant from v by $\sqrt{3}$ unit distance. This is clearly demonstrated in Fig 5 (c) by means of a central red color voxel p. Fig 5 (c) represents the 26-neighborhood property of the neighborhood. The neighborhood of all voxels of v fulfills the following condition [3][7]:

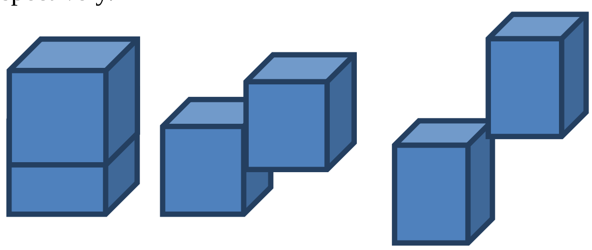
$$N_{26}(v) = N_6(v) \cup N_{12}(v) \cup N_8(v) \quad (7)$$

$$1 \le |x_i - x_j| + |y_i - y_j| + |z_i - z_j| \le 3 \quad (8)$$

(or)

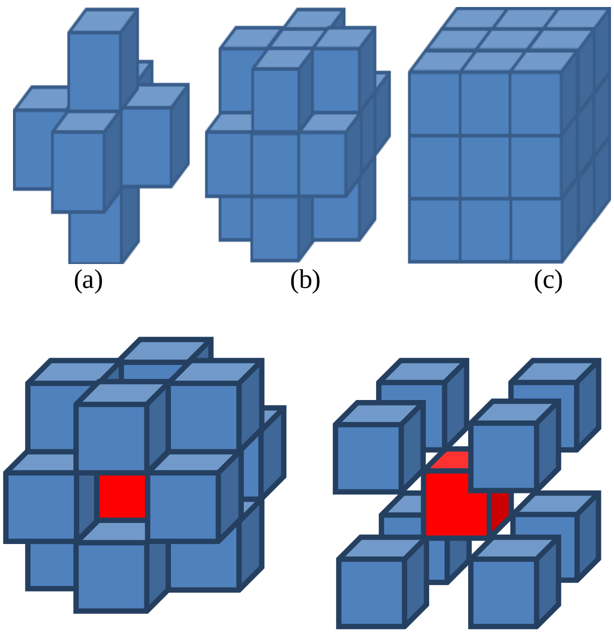
$$N_{18}(v) = S_1 \cup S_2 \cup S_3 \quad (9)$$

To present differently, 6-neighborhood, 12-neighborhood, and 8-neighborhood are the neighborhoods which share the faces, edges, and vertices with other cubes as demonstrated in Fig 7 (a), Fig 7 (b), and Fig 7 (c), respectively.



(a) (b) (c)
(a)Face Neighbor; (b) Edge Neighbor; (c) Vertex Neighbor
Fig 7: 3-D neighborhoods

Fig 8 represents convex structuring elements in (a), (b) & (c) and concave structuring elements in (d) & (e).



(a) (b) (c)
(d) (e)

Fig 8: (a) 6-Neighborhood (b) 18-Neighborhood (c) 26-Neighborhoods (d) 12-neighborhood (e) 8-neighborhood

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Adjacency
A pair of voxels in a $\mathbb{C}_n^k$ are k-cells, whose adjacency is decided by the m-cell. A m-cell is a k-cell of an m-dimensional facet [3].

Two k-cells are called m-adjacent if they share a common m-cell.

A voxel u is said to be m-adjacent to a voxel v if it belongs to Nm(v), where m= 6,8,12,18,26.

Most frequently used adjacencies in 3-D digital images are:

- 6-adjacency
- 12-adjacency
- 8-adjacency
- 18-adjacency
- 26-adjancecy

Let us consider a voxel pair (u,v) where u=(xi,yi,zi) v=(xj,yj,zj) . A voxel v is said to be m-adjacent in place if it belongs to any one of the following:

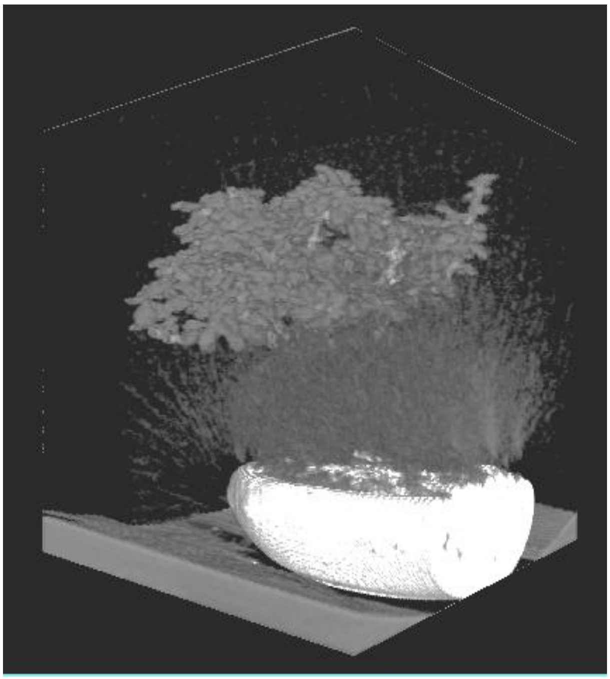
- 6-adjacency [A6(v)]: A voxel u is said to be 6-adjacent to a voxel v if it belongs to 6-neighborhood N6(v).
- 12-adjacency [A12(v)]: A voxel u is said to be 12-adjacent to a voxel v if it belongs to 12-neighborhood N12(v).
- 8-adjacency [A8(v)]: A voxel u is said to be 8-adjacent to a voxel v if it belongs to 8-neighborhood N8(v).
- 18-adjacency [A18(v)]: A voxel u is said to be 18-adjacent to a voxel v if it belongs to 18-neighborhood N18(v).
- 26-adjacency [A26(v)]: A voxel u is said to be 26-adjacent to a voxel v if it belongs to 6-neighborhood N26(v).

IV. USE OF NEIGHBORHOOD IN THE PROCESSING OF 3-D IMAGES

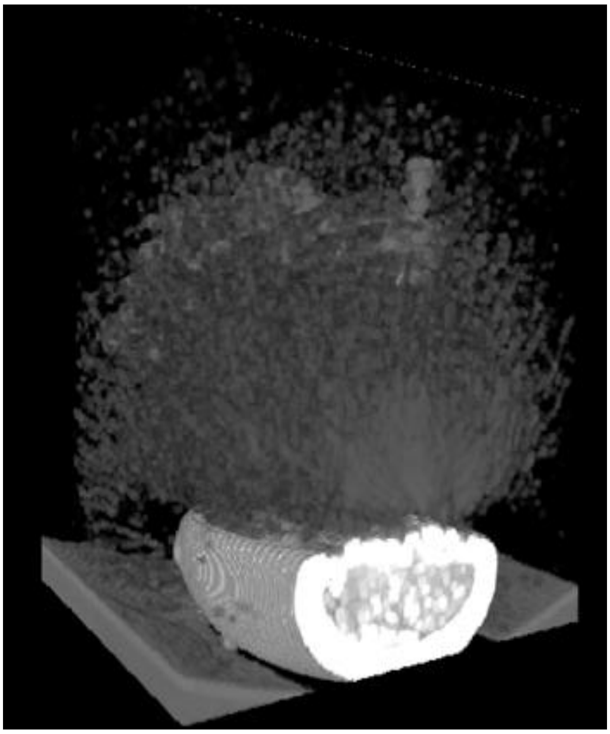
A 3-D digital image is examined here as a configuration of a set of neighborhoods. The convolution operation is performed on the original 3-D image of size NxNxN with a window of size of MxMxM. In general the window size is a odd number, for example M=3,5,7,9..., i.e. 3x3x3 etc. Within the 3x3x3 window size there might be 256 convex polyhedrons[5] [7] [8]. So there are 256 different shapes in a convex polyhedrons. By choosing different shape for a window in a convolution operation, there will a different resultant in a processed image.

The given 3-D digital image is plane-wise raster-scanned by the 6/8/12/18/26-neighborhood window. Upon every move, the 3x3x3 sub image embedded by this window is investigated to verify the particular property in the original 3-D image. Such operations are performed till the entire image is scanned. The figure 9(a) shows a original 3-D image of a Bonsai and figure 9(b) & 9(c) shows dilated & eroded 3-D image with 6-neighborhood window/structuring element in the mathematical morphology framework, respectively.

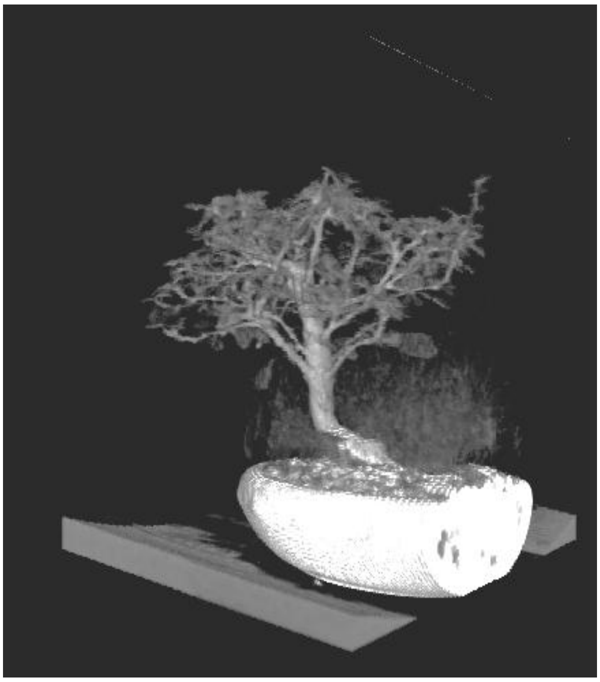
Hence it is concluded that by changing the various neighborhood structures, the resultant processed 3-D images would be different. But the differently resulted images are barely differentiated by the human eye.



(a)



(b)



(c)

Fig 9: (a) Original Image (b) Dilated Image (c) Eroded Image

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BIOGRAPHIES

Mr. K. Rajiv did his M.Tech in Computer Science and Engineering from SRKR Engg. College, Bhimavaram. Presently. He is working as Assistant Professor in the Department of Information and Technology, Sridevi Women's Engineering College, Hyderabad. He is currently pursuing his PhD in Acharya Nagarjuna University, Guntur. He published 5 papers in National and International Journals and 3 papers in International Conferences. He has 6 years of Teaching Experience.

Dr. G. Ramesh Chandra did his PhD in the area of image processing. He has 15 Years of teaching and research experience. His areas of interest are Image processing, Video Processing and Data Mining. He actively participated in Research and Consultancy Centre (RCC) Projects. In the year 2013 he become Head of RCC. He published 20 Research Papers in National and International Journals and Conferences. He has 15 years of Teaching Experience.

Dr. B. Basaveswara Rao did his Ph.D in Acharya Nagarjuna University in the area of Computer Networks. He is working as Web Administrator in Computer Center, Acharya Nagarjuna University. His areas of interest are Computer Networks, Network Security and Test Mining. He published 10 papers in National and International journals. He has 25 years of Teaching Experience.